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Advanced Algorithms, Fall 2007

Assignment #4
Due: Nov 13th, 2007

Each of the problems should be solved on a separate sheet of paper to facilitate grading. Please don't wait until the last minute to look at the problems.

Problem 1 A slab is a region between two parallel lines. The width of a point set P, denoted by $w(P)$ is the width of the smallest slab that contains all the points in P. Show that $w(P) = w(\text{ConvexHull}(P))$. Compute the width of P in linear time given the convex hull of P in counter-clockwise order.

Problem 2 In the proof of the query time of the kd-tree we found the following recurrence:

$$Q(n) = \begin{cases} O(1) & \text{if } n = 1 \\ 2 + 2Q(n/4) & \text{if } n > 1 \end{cases}$$

Prove that the recurrence solves to $Q(n) = \sqrt{n}$. (Use of master's theorem is prohibited)

Problem 3 Show that the dual of the dual of a linear program is the original program itself.

Problem 4 In many applications one wants to do range searching among objects other than points.

a. Let S be a set of n axis-parallel rectangles in the plane. We want to be able to report all rectangles in S that are completely contained in a query rectangle. Describe a data structure using $O(n(\log n)^3)$ storage and with $O((\log n)^4 + k)$ query time where k is the number of reported answers. Hint: use orthogonal range search in a higher-dimensional space.

b. Let P consist of a set of n polygons in the plane. Again, describe a data structure using $O(n(\log n)^3)$ storage and with $O((\log n)^4 + k)$ query time where k is the number of reported answers.

Problem 5

a. Let R be a set of n red points in $\Re^2$, and let B be a set of n blue points in $\Re^2$. We call a line l a *separator* for R and B if l has all points of R to one side and all points of B to the other side. Give a randomized algorithm that can decide in $O(n)$ expected time whether R and B have a separator.

b. Let $P := \{p_1, p_2, \ldots, p_n\} \in \Re^2$. Let $p \in P$ be one of these points. Give a randomized algorithm that can decide in $O(n)$ expected time whether p is a vertex of the $CH(P)$.